Complexity of maintaining (canonical) tree-like decompositions

Frank Fuhlbrück

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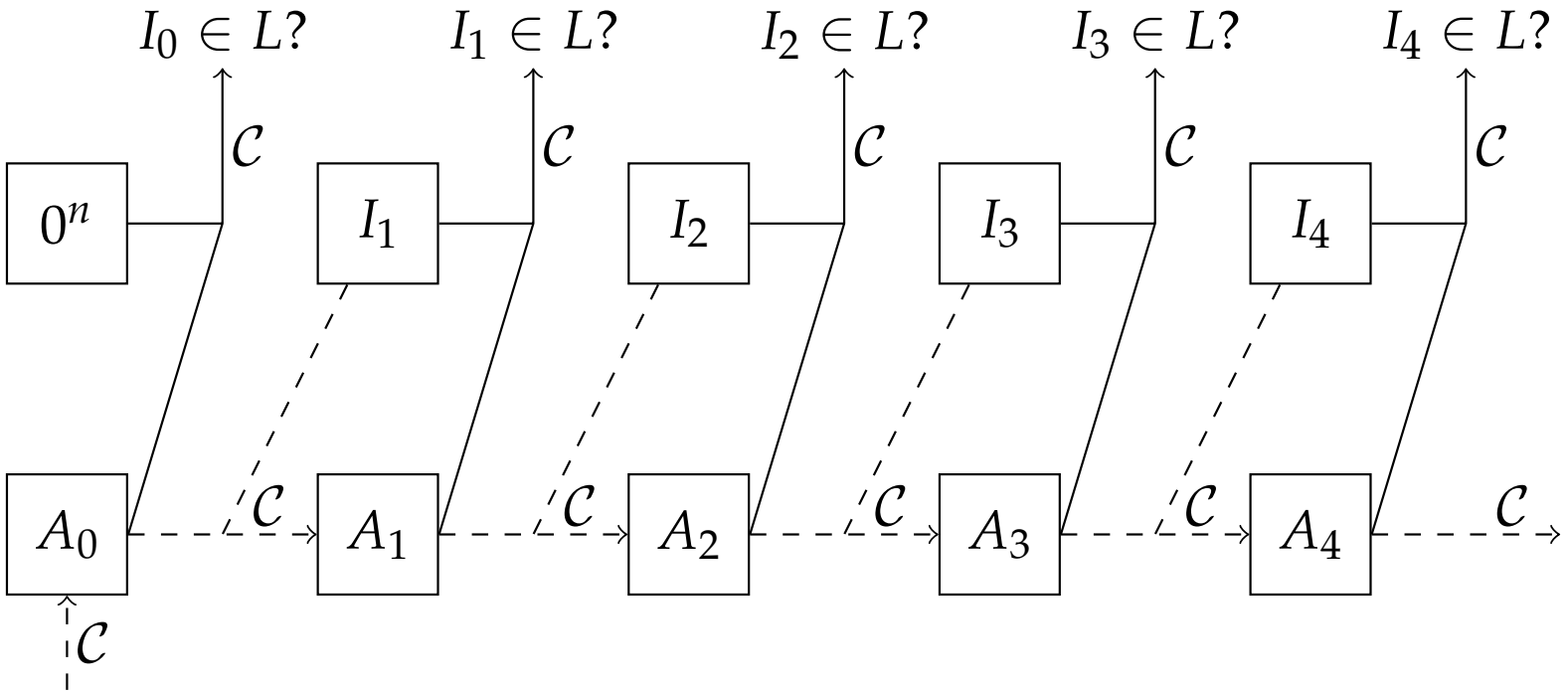
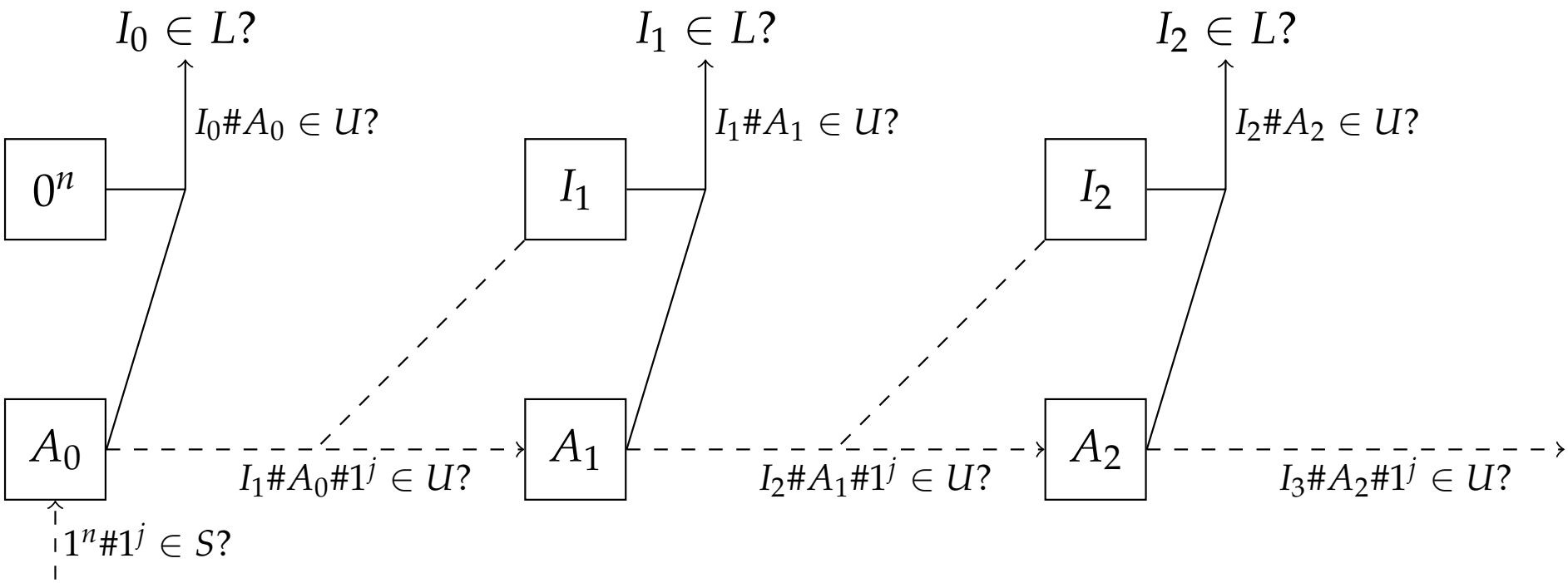
- The auxiliary information stems from a series of instances $I_0, \ldots, I_l = I$ differing only slightly, i.e. hamming distance $\Delta(I_i, I_j) = 1$ (strings) or a single tuple inserted/removed (structures).
- For graphs: a single edge inserted or deleted

Dynamic complexity illustrated



I_i - Instances, $\Delta(I_i, I_{i+1}) = 1$
A_i - Auxiliary information

Dynamic complexity illustrated - details



$\{U, S\} \subseteq \mathcal{C}$

I_i - Instances, $\Delta(I_i, I_{i+1}) = 1$

A_i - Auxiliary string:

jth bit of A_i: $A_{i,j} = 1 \Leftrightarrow I_i\#A_{i-1}\#1^j \in U$

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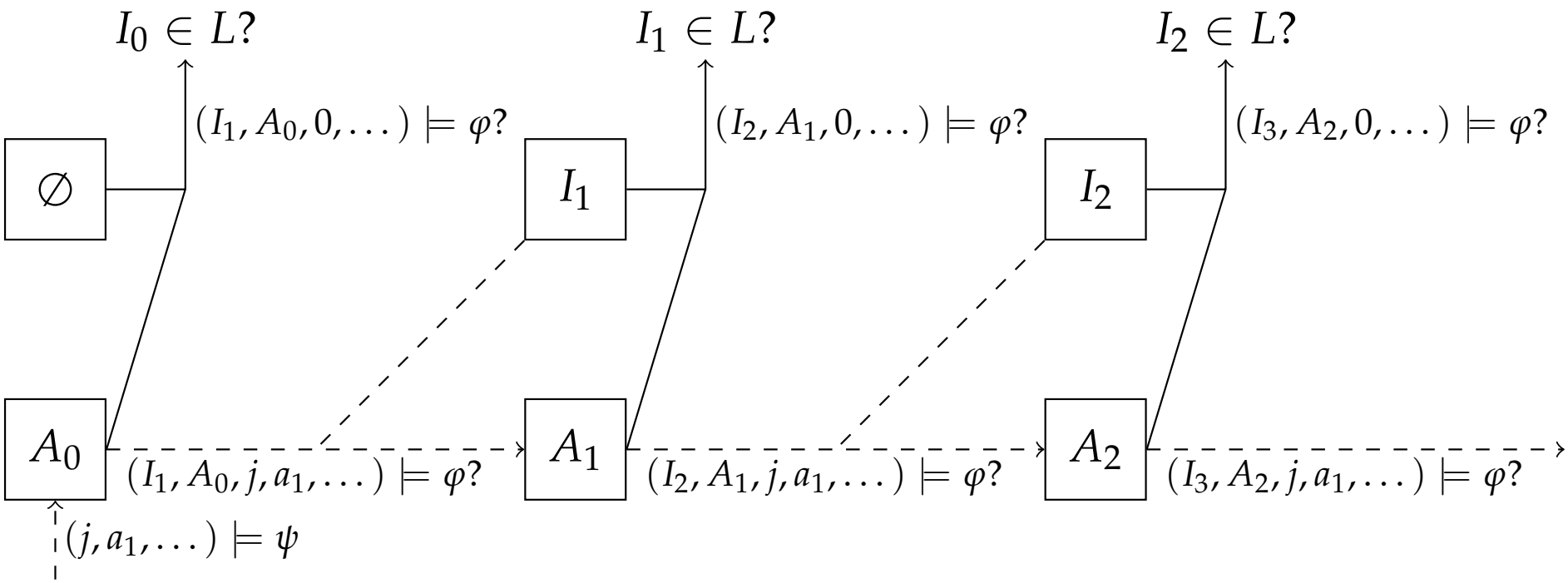

$$\{\varphi, \psi\} \subseteq \Phi$$

I_i - Instances, one tuple changed from I_i to I_{i+1}

A_i - Auxiliary structure:

$$(a_1, \dots, a_m) \in R_j^{A_i} \Leftrightarrow (I_i, A_{i-1}, j, a_1, \dots, a_m, \dots) \models \varphi$$

Important dynamic classes

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- further interesting: DynNC^i, DynL, ...
- but DynP is just P

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- e.g. there are problems in DynFO that are P-complete under first order reductions [MSVT94; PI97]
- therefore, we define:

Definition

$A \leq_{b\mathcal{C}} B$ if there is an $A \leq_{\mathcal{C}} B$ reduction function f with

$$\forall c \exists c' : \Delta(I, J) \leq c \Rightarrow \Delta(f(I), f(J)) \leq c' .$$

That means every input bit influences only a constant number of output bits.

Known Results for Dynamic Canonization

- canonization of trees is in DynFO [Ete98]

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- canonization of planar 3-connected graphs in DynFO^+ (polynomial time precomputation) [Meh13]

Tree-depth

Definition ([NO06; BDK12])

Let G be a graph with connected components $C_1, \ldots, C_l$. Then

$$\mathrm{td}(G) = \begin{cases} 1 & |V(G)| = 1 \\ \max_{i \in [l]} \mathrm{td}(G[C_i]) & l > 1 \\ 1 + \min_{v \in V(G)} \mathrm{td}(G - v) & \text{else} \end{cases} .$$

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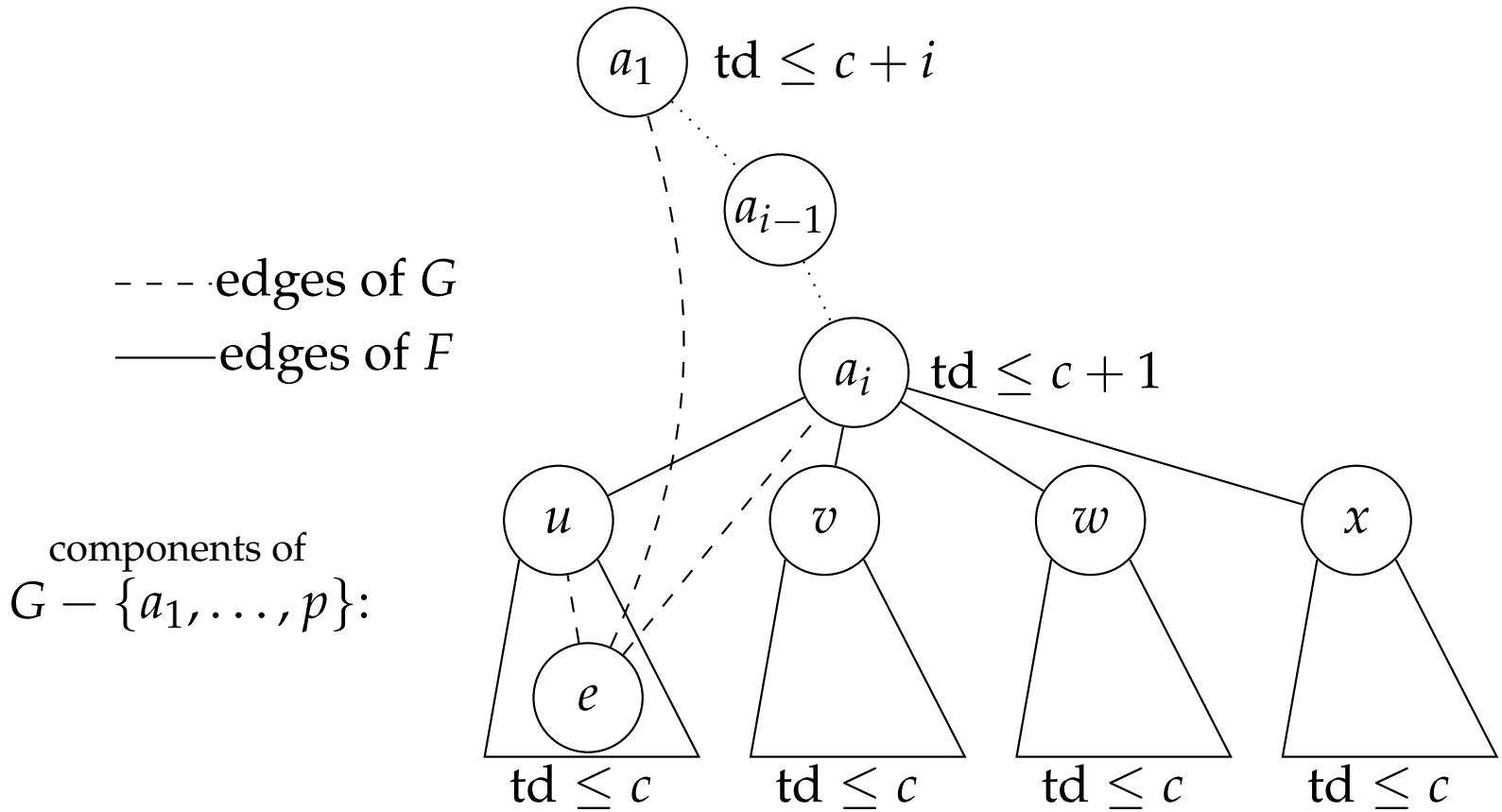
A rooted forest $(F, r_1, \ldots, r_l)$ is a tree-depth decomposition of G if $V(G) = V(F)$ and

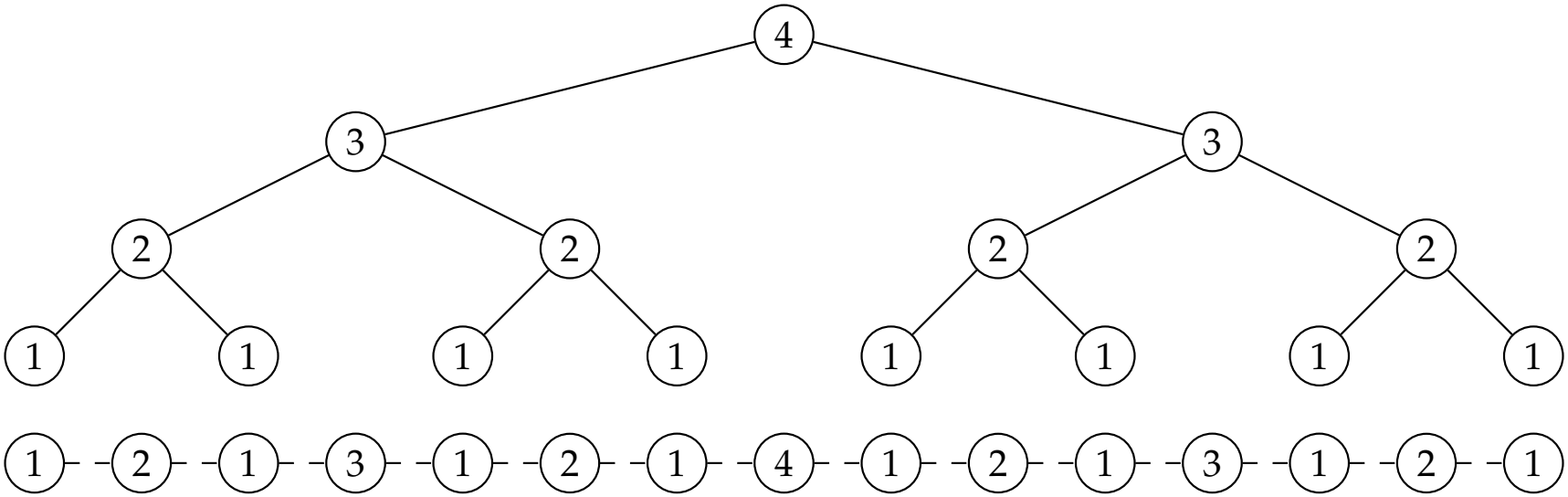
$$\begin{cases} r_1 = v & V(G) = \{v\} \\ \forall 1 \leq i \leq l : (F[C_i], r_i) \text{ is a tree-depth dec. of } G[C_i] & l > 1 \\ (F - r_1, r'_1, \ldots, r'_{l'}) \text{ is a tree-depth dec. of } G - r_1 & \text{else} \\ \text{where } N_F(r_1) = \{r'_1, \ldots, r'_{l'}\} \end{cases}.$$

Tree-depth decomposition - illustrated



Every neighbor of e in G lies on path from e to a_1 in F.

Tree-depth decomposition of a path



A path of length 14 has tree-depth 4.

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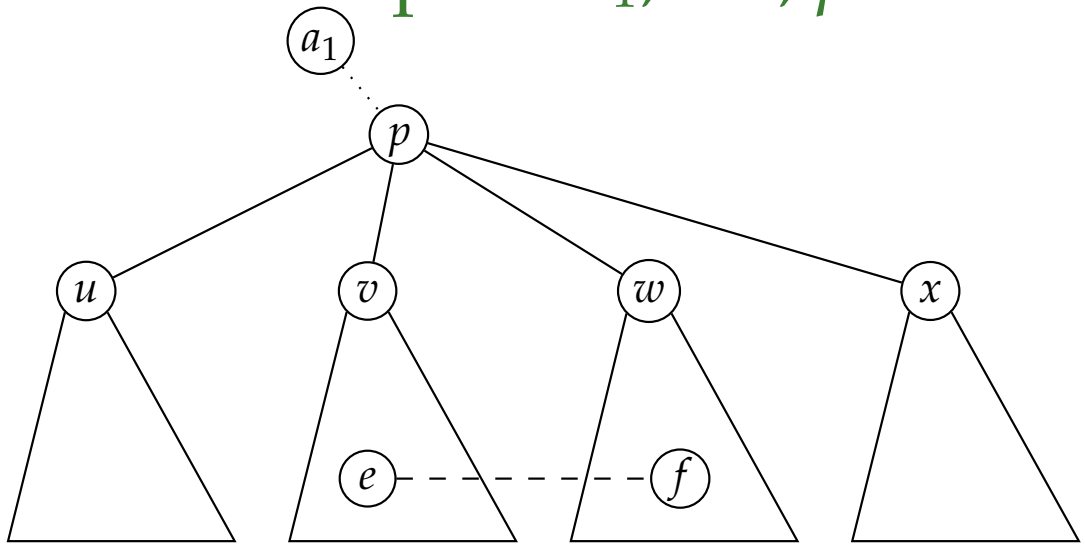
≤ 2 components below a fixed path $a_1, \ldots, p$

canonical dec. with root path $a_1, \ldots, p, \ldots$



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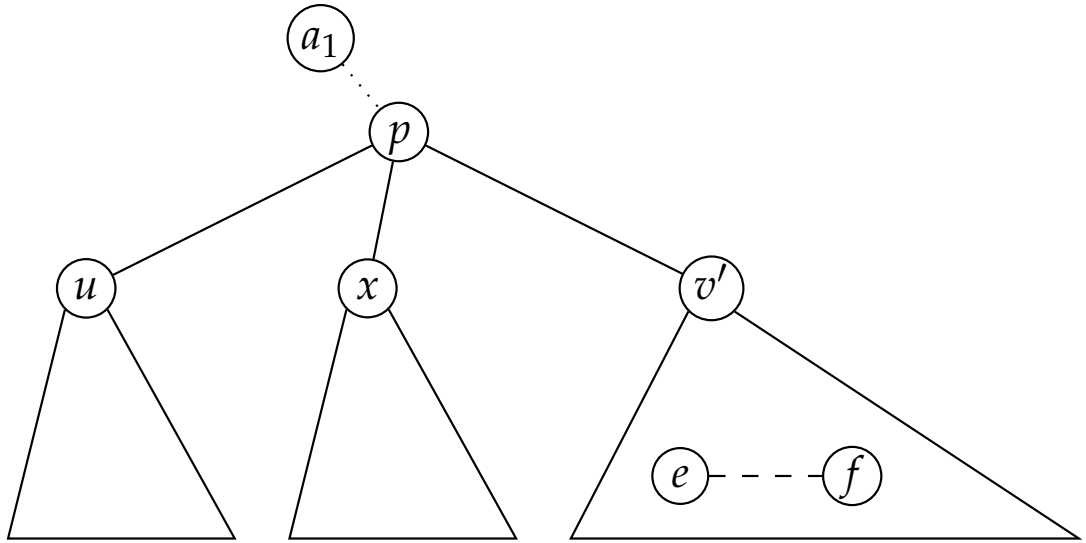

Using a canonical tree-depth dec. for isomorphism

- uniquely color every vertex a_i with the child indices of its ancestors $a_1, \ldots, a_{i-1}$ in the tree-depth dec. (like in [Meh13], but simpler)

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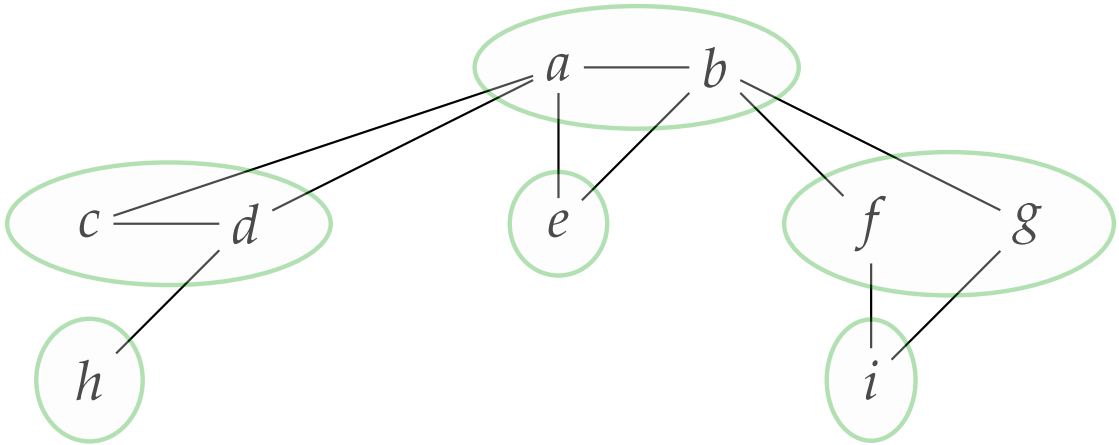
Theorem

GraphIso for graphs of bounded tree-depth is in DynFO (*and* TC^0).

Tree distance decompositions

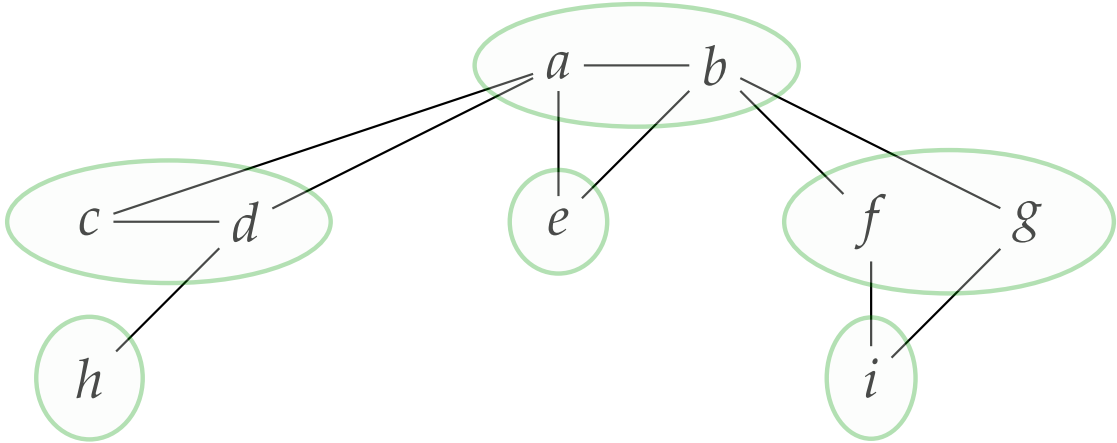


Definition

Let G be a graph. A (*minimal*) *tree distance decomposition* of G is a triple (B, T, r), where

1. T is a tree and $r \in V(T)$,
2. $B : V(T) \to \wp(V(G))$ such that $\mathrm{rng}(B)$ is a partition of $V(G)$ and
3. for all $e \in E(G)$ $\exists t, u \in V(T)$ such that $e \subseteq B_t \cup B_u$ and $t = u$ or $\{t, u\} \in E(T)$ and

Tree distance decompositions



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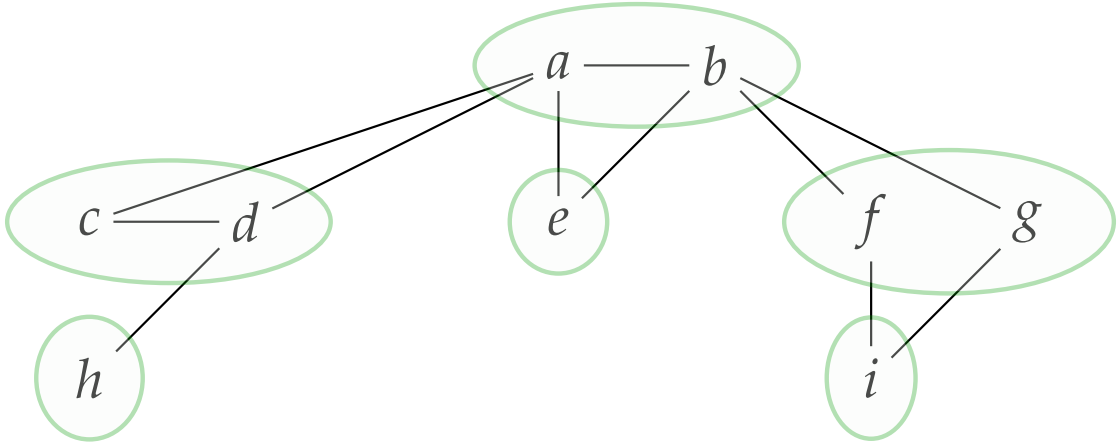
4. for all $t \in V(T)$ and all $v \in B_t$: $\min_{u \in B_r} \mathrm{d}(u, v) = \mathrm{d}(r, t)$ and

Tree distance decompositions



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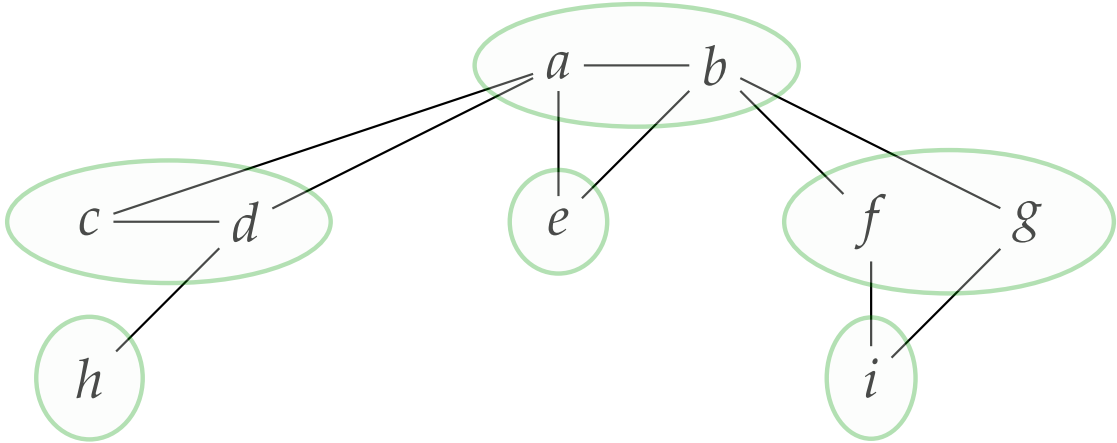
5. (minimality) for all $t \in V(T)$: $G[\bigcup_{t' \in V(T_t)}]$ is connected, where T_t is the subtree rooted at t.

Tree distance decompositions



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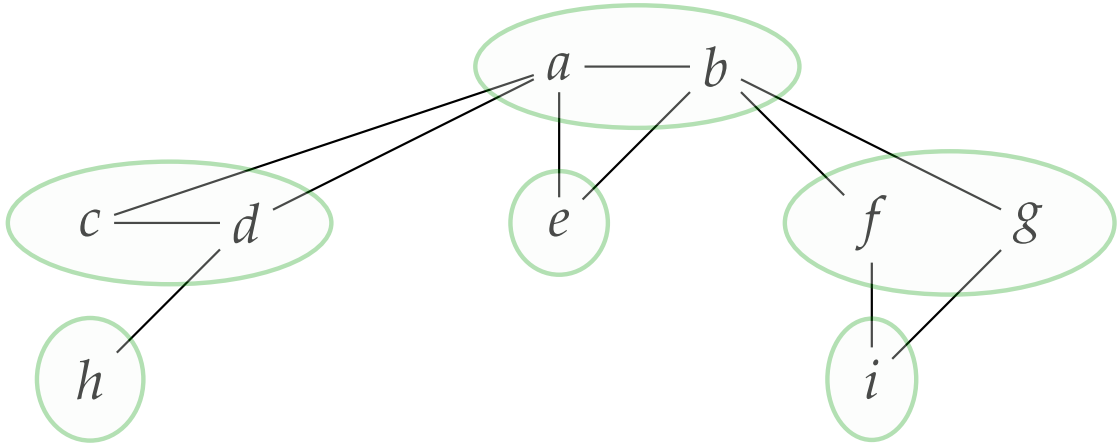
It is a *path distance decomposition* if T is a path.
The *width* of a tree distance decomposition is $\max_{t \in V(T)} |B_t|$. The *path/tree distance width* $\mathrm{pdw}(G)/\mathrm{tdw}(G)$ is the minimal width of a path/tree distance decomposition of G.

Maintaining a minimal tree distance decomposition

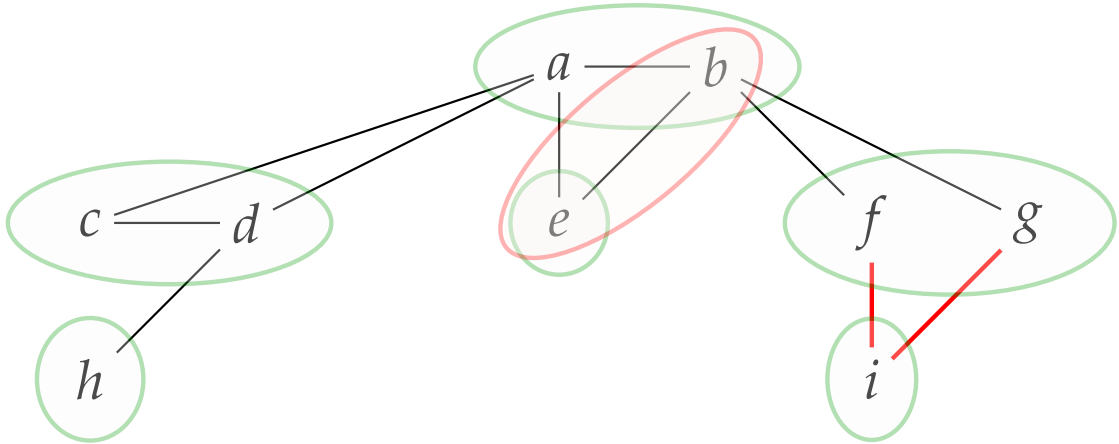


What constitutes a bag w.r.t. to root bag $R = \{a, b\}$?

We need a predicate that tells us for every k-element set (henceforth "bag") whether it is a bag.

Computing pairwise distances is in DynFO [Meh13] and and the minimum can be expressed in first order.

Maintaining a minimal tree distance decomposition



What constitutes a bag w.r.t. to root bag $R = \{a, b\}$?

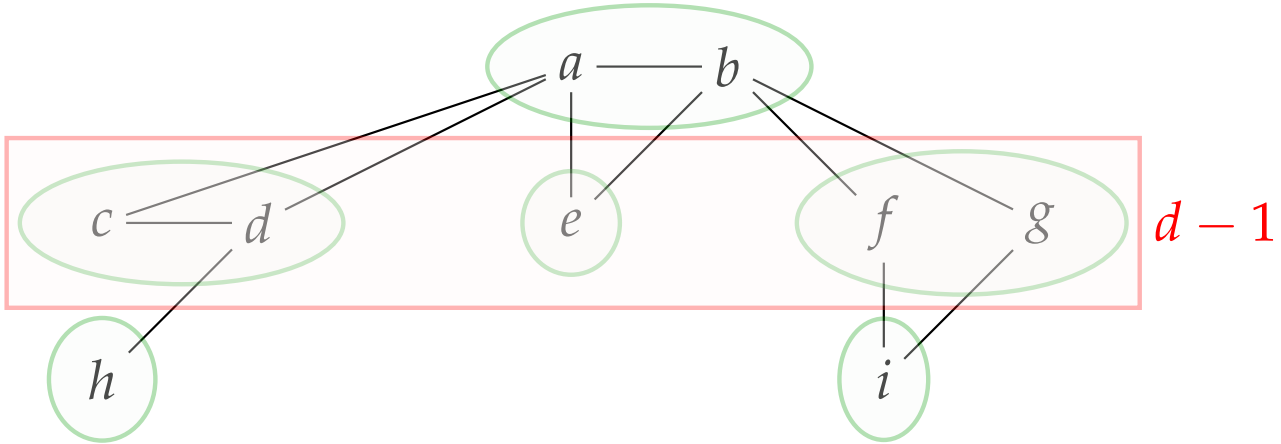
1. What is in the same component below some "bag" P?

$\text{inComp}_{R,P}(u, v) \Leftrightarrow u$ is reachable from v

in $G - P$ but from no vertex in R

e.g. $\text{inComp}_{\{a,b\},\{b,e\}}(f, g)$ holds

Maintaining a minimal tree distance decomposition



What constitutes a bag w.r.t. to root bag $R = \{a, b\}$?

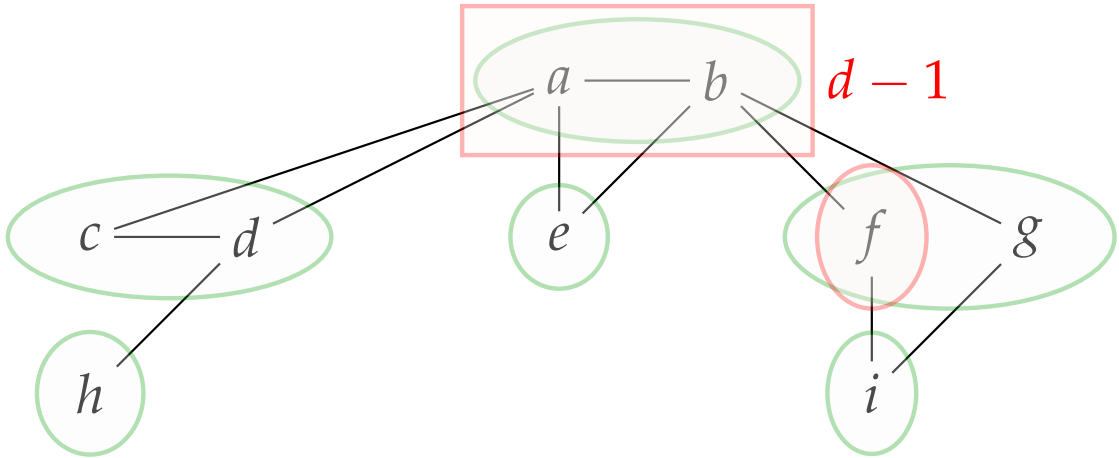
2. What is in one comp. below all "bags" with distance $d - 1$?

$$\text{inCompLvl}_{R,d}(u, v) \Leftrightarrow \forall B, |B| \leq k \wedge$$

$$v \in B \Rightarrow d(R, v) = d - 1 : \text{inComp}(R, B, u, v)$$

e.g. $\text{inCompLvl}_{\{a,b\},2}(h, i)$ does not hold

Maintaining a minimal tree distance decomposition



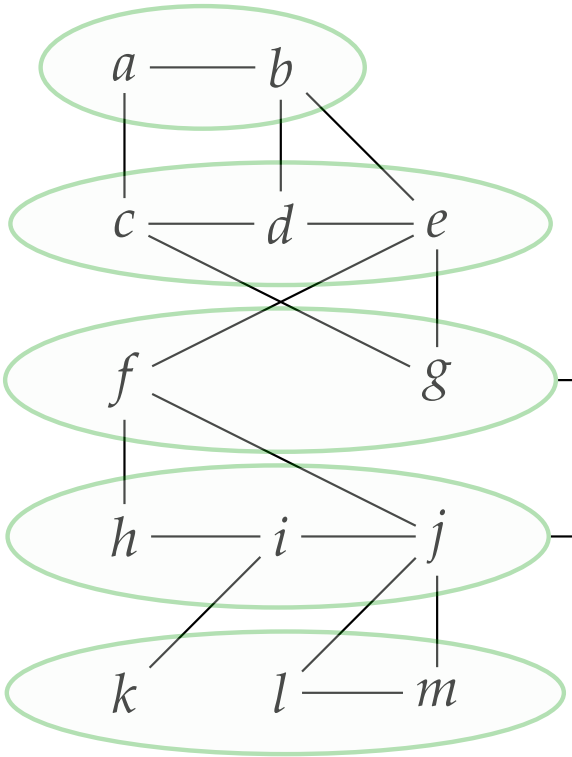
What constitutes a bag w.r.t. to root bag $R = \{a, b\}$?

3. Which "bags" are really bags?

$$\begin{aligned}\text{isBag}(R, B) \Leftrightarrow \exists d : (\forall v \in B : d(R, v) = d) \wedge \\ (\forall u, v \in B : \text{inCompLvl}(R, d, u, v)) \wedge \\ \forall u : (\exists v : v \in B, d(R, u) = d, \text{inCompLvl}(R, d, u, v)) \Rightarrow u \in B\end{aligned}$$

e.g. $\text{isBag}_{\{a,b\}}(\{f\})$ does not hold, because g is reachable from f in $G - \{a, b\}$

Canonization of PDDs in NC^1



$$L_{i,i+1} = \{(\varphi, \psi) \mid \exists \text{lab.} \chi \text{ of } G[B_i \cup B_{i+1}] : \varphi = \chi|_{B_i}, \chi \text{ is minimal s.t. } \chi|_{B_{i+1}} = \psi\}$$

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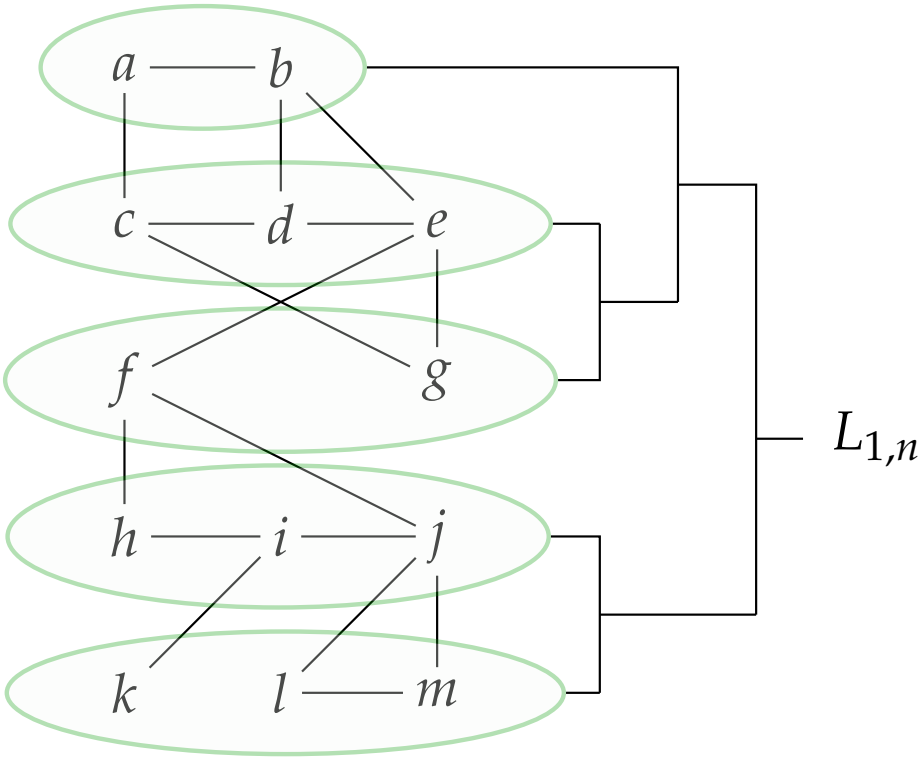
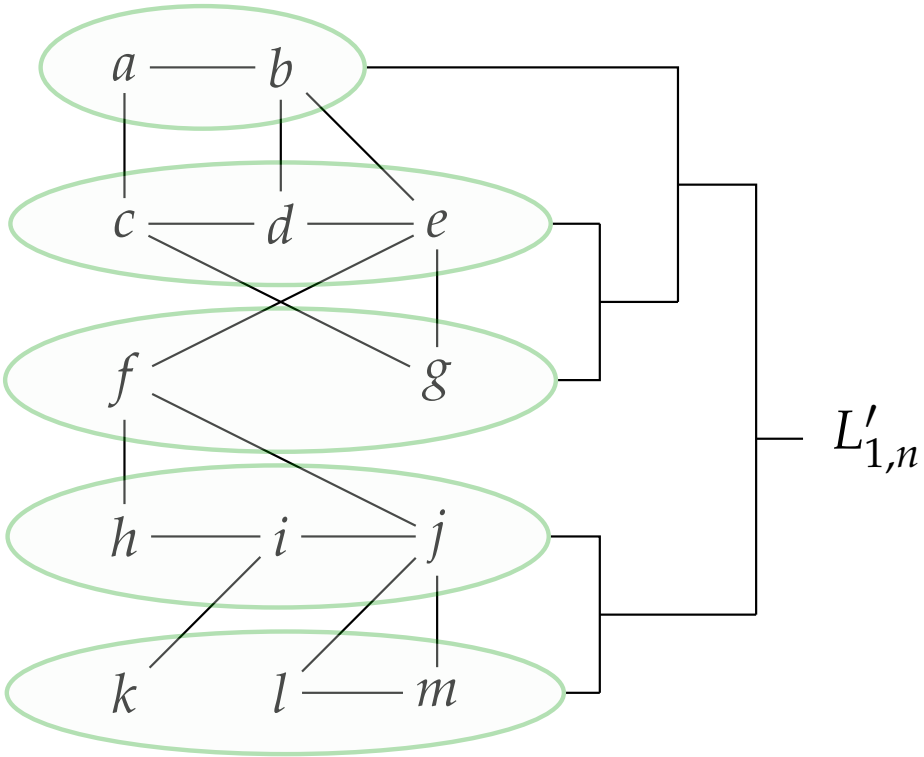


$$L_{i,j} = \{(\varphi, \psi) \mid \exists (\varphi, \psi') \in L_{i,k}, (\varphi', \psi) \in L_{k+1,j} : (\psi', \varphi') \in L_{k,k+1}\}$$

Canonization of PDDs in NC^1



$$L'_{i,j} = \left\{ (\varphi, \psi) \mid \exists (\varphi, \psi') \in L'_{i,k}, (\varphi', \psi) \in L'_{k+1,j} : (\psi', \varphi') \in L_{k,k+1} \right\}$$

where $L'_{i,i+1}$ depends on $L_{i,n}$ and $L_{i+1,n}$

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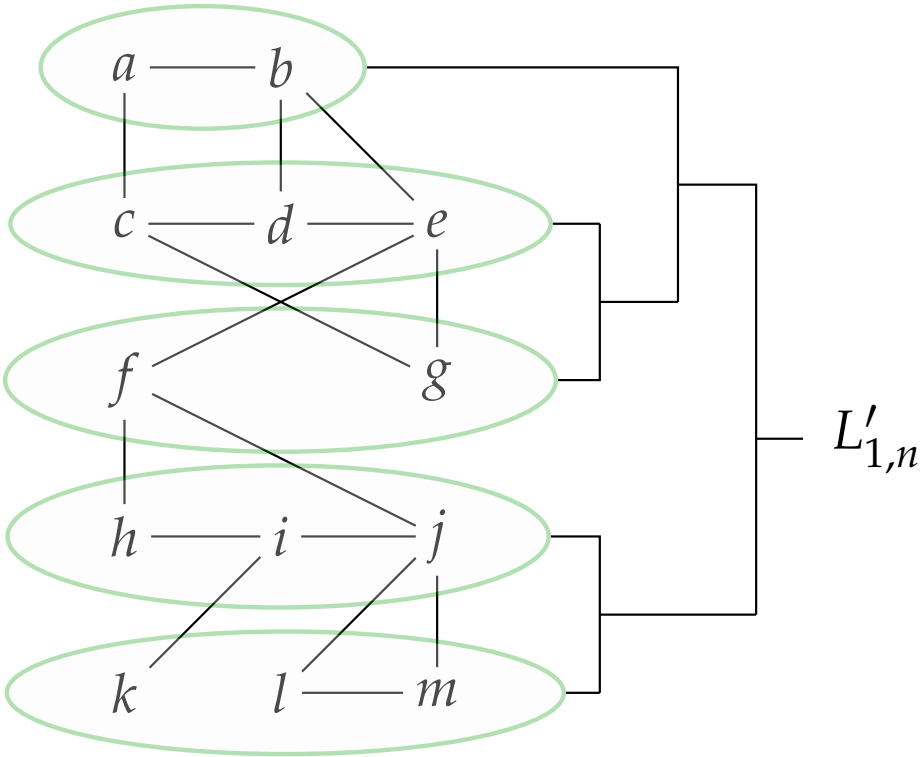

Now we choose $\varphi_1 \in \mathrm{dom}(L'_{1,n})$
and *independently* one φ_i from each $L'_{1,i}(\varphi_1)$.

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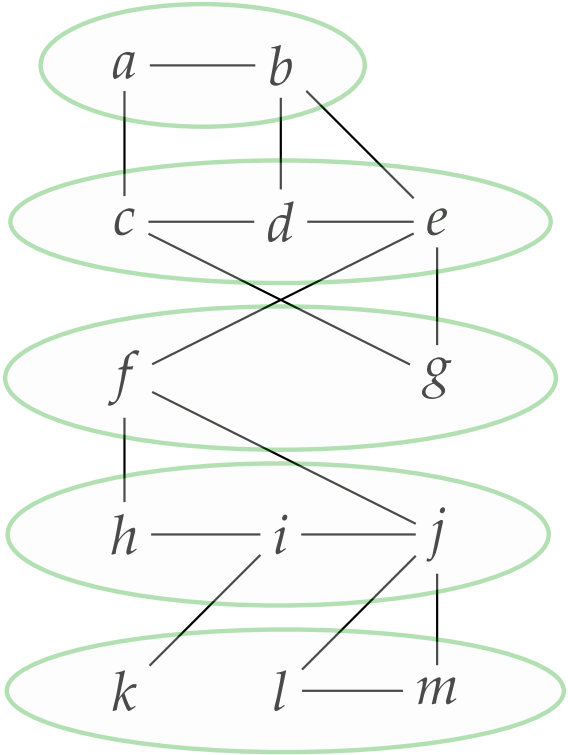

Theorem

GraphIso for graphs of bounded path-distance width is in DynNC^1.

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- thus this gives us a bounded first order reduction to tree canonization [Ete98]

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  - For bounded treewidth in DynL? (and later in L?)

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